CUSTOMER SUPPORT NOTE

Performing a Buckling Analysis

Note Number:	**CSN/LUSAS/1019**

This support note is issued as a guideline only.

LUSAS

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1. Introduction

Linear elastic analysis is suitable for structures where geometric nonlinear effects (e.g. buckling) and material nonlinear effects (e.g. yielding or cracking) have little or no impact on the structural response. Results from a linear elastic analysis can also be used in simplified calculation methods, typically provided in design codes, that account for geometric and material nonlinearities. However, there are many cases where a more detailed analysis is required.

This support note describes three methods available in LUSAS for analysing the buckling behaviour of a structure. These are:

- Linear Eigenvalue Buckling Analysis (LBA)
- Geometrically Nonlinear Analysis (GNA)
- Geometrically and Materially Nonlinear Analysis (GMNA)

These methods are described in more detail below. The second and third can also account for nonlinear boundary conditions, where necessary.

A geometrically nonlinear analysis with initial imperfections is abbreviated as GNIA, while a geometrically and materially nonlinear analysis with initial imperfections is abbreviated as GMNIA.

2. Example – Structural model

In this example, a bridge comprising two steel plate girders is considered, as shown in Figure 1. Transverse bracing is provided to enhance the global lateral stability of the girders. Intermediate transverse stiffeners are used to improve the stability of the girder webs. The four bearings are idealised as line supports in the numerical model.

The girders, including the top and bottom flanges, webs, and stiffeners, are modelled using thick shell elements to predict both global and local buckling modes. The transverse bracing is modelled using thick beam elements. Both girders are subjected to uniformly distributed vertical loads.

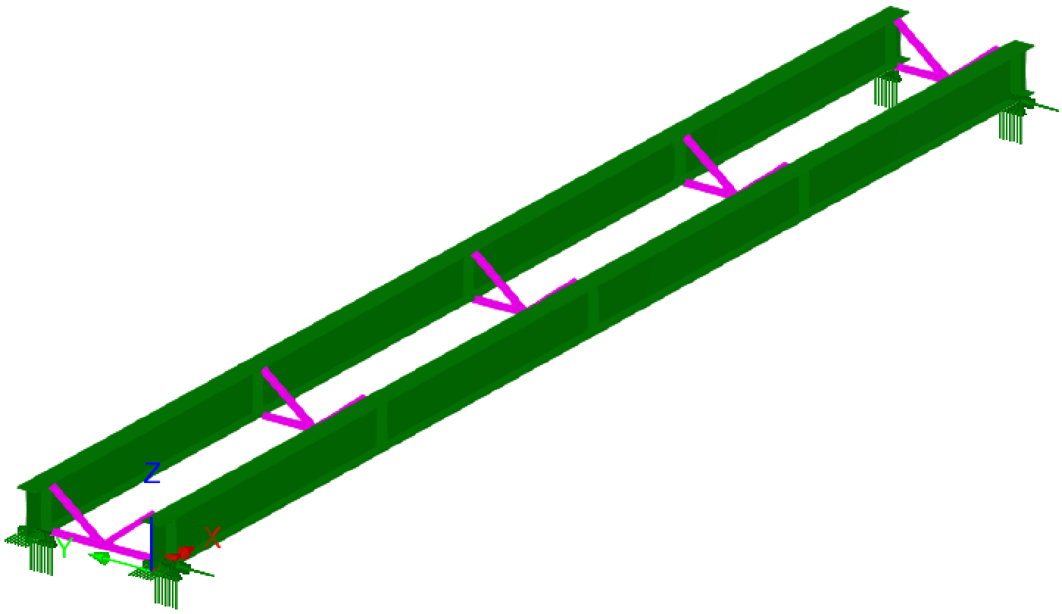

Figure 1 – 3D model of bridge.

Three types of analysis are performed: eigenvalue buckling analysis, geometrically nonlinear analysis, and geometrically and materially nonlinear analysis.

3. Linear (Eigenvalue) Buckling Analysis

3.1 General information

An eigenvalue buckling analysis provides:

1. Buckling mode shapes.
2. The corresponding buckling load factors.

An eigenvalue buckling analysis assumes linear elastic behaviour and does not account for geometric or material nonlinearities, initial imperfections, residual stresses, or other effects that may influence the actual buckling behaviour of a structure. Consequently, the predicted critical buckling load is generally considered an upper-bound estimate of the buckling capacity. However, when geometric and material nonlinearities, initial imperfections, and other relevant effects are negligible, an eigenvalue buckling analysis may provide a close approximation to the actual load at which buckling occurs.

The fundamental assumptions of an eigenvalue buckling analysis are:

- The linear stiffness matrix remains constant before buckling occurs.
- The stress stiffness matrix is a scaled version of the stress stiffness matrix corresponding to the reference load.

The Help documentation provides further information on the types of elements that can be used in an eigenvalue analysis.

An eigenvalue buckling analysis is not expected to provide accurate results when pre-buckling deformations are significant. Consider the member shown in Figure 2a. The member buckles under a compressive axial force, as shown in Figure 2b. However, when a horizontal load is also applied, as shown in Figure 2c, it causes pre-buckling deformations that are not accounted for in the eigenvalue analysis. As a result, the predicted buckling load may not be accurate.

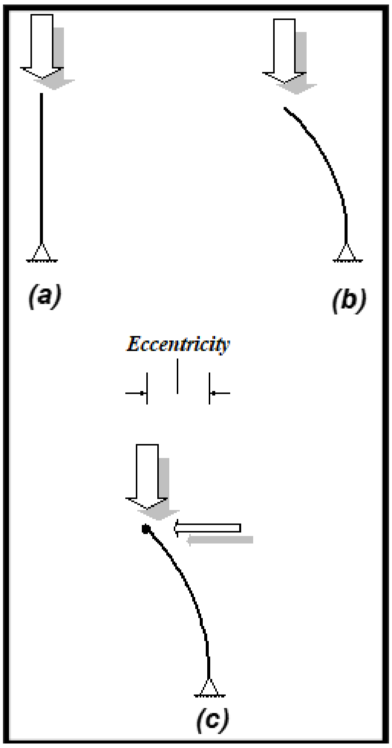

Figure 2 – (a) – (b) Axially loaded member and (c) axially and laterally loaded member.

It is also worth noting that an eigenvalue buckling analysis does not capture post-buckling behaviour; a nonlinear buckling analysis is required to predict the post-buckling response of a structure.

An eigenvalue buckling analysis can capture both local and global buckling modes. Engineering judgement is essential for determining which modes are significant and require further investigation.

3.2 Example – Setting up an LBA

In this example, Analysis 1 is set up as an eigenvalue buckling analysis. To set it up:

- In the Analyses treeview, expand Analysis 1, right-click the loadcase listed under it, and select Controls > Eigenvalue.
- Set Solution to Buckling load.

LUSAS provides a range of eigensolver types to choose from. Further information on the available solvers can be found in the *LUSAS Solver Manual*.

In this example, five eigenvalues are calculated using the default eigensolver with its standard settings (Figure 3).

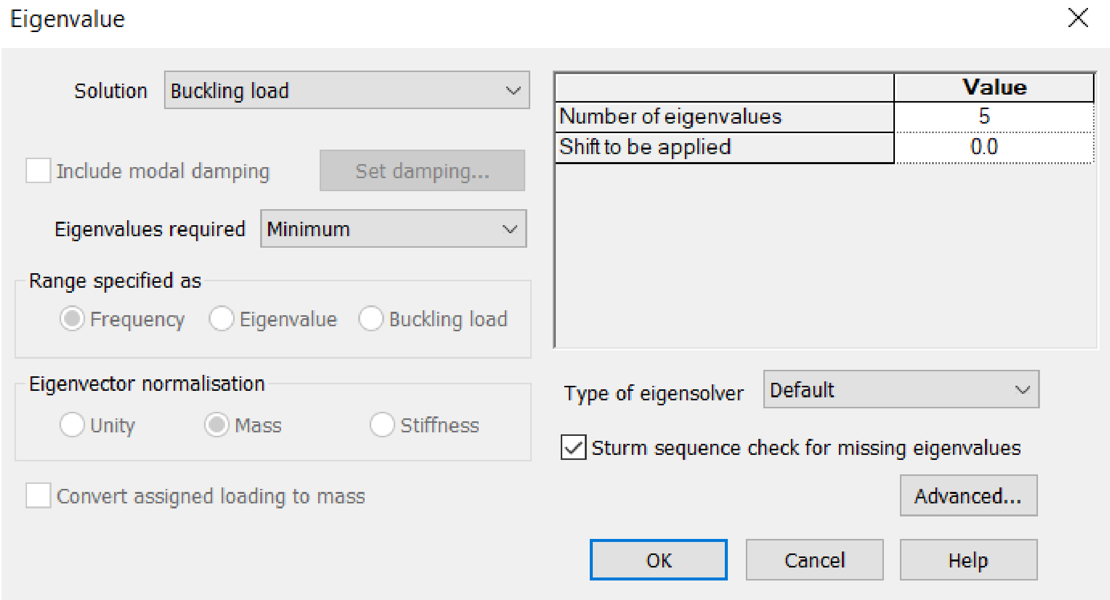

Figure 3 – Eigenvalue controls.

3.3 Example – Results

The results are presented in Figure 4 (Utilities > Print Results Wizard > Eigenvalues > Loadcases: Active > Eigenvalues (Load Factor)).

The second eigenvalue is negative. When negative eigenvalues are obtained, LUSAS generates warning messages in both the Text Window and the output file (see Technical Note 1030: Negative Eigenvalues). It is always important to check that the eigenvalues of interest are computed with sufficiently small error norms.

	Mode ▲	Eigenvalue	Load Factor	Error norm
1	1	1.59664	1.59664	0.634706E-6
2	2	-1.80222	-1.80222	6.06693E-9
3	3	1.8205	1.8205	0.204251E-9
4	4	2.14157	2.14157	28.3839E-9
5	5	2.1604	2.1604	0.378673E-6

Figure 4 – Eigenvalue analysis results.

In this example, the load factor corresponding to the first buckling mode is 1.5966, indicating that the applied load must be increased by a factor of 1.5966 to reach the critical buckling load.

The first buckling mode shape is shown in Figure 5. The mode shape represents the relative distribution and direction of deformation; the displacement values depend on the chosen normalisation and should not be interpreted as the displacements that would occur at the corresponding buckling load.

The force and reaction outputs are derived from the displacements associated with the eigenvectors (mode shapes). Consequently, these outputs represent the forces and reactions that would result if the corresponding eigenvector displacements were imposed on the structure as prescribed displacements. To determine the forces and reactions at the buckling load, a separate linear elastic analysis needs to be performed using the original loading pattern, scaled by the load factor obtained from the eigenvalue buckling analysis.

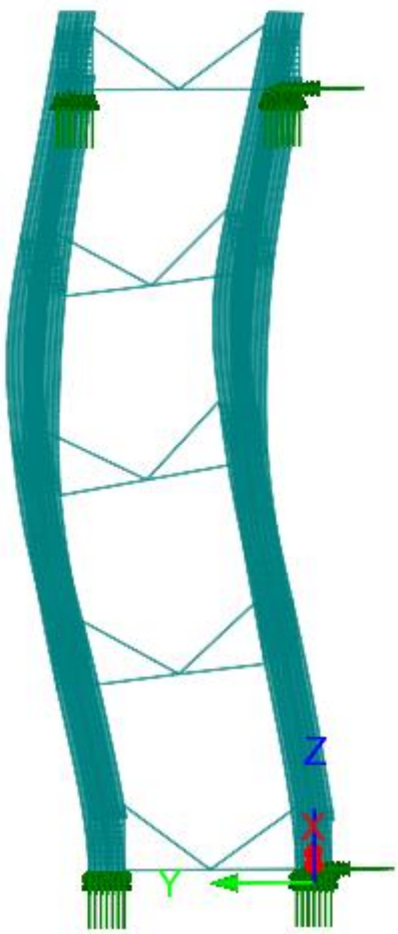

Figure 5 – First buckling mode shape.

4. Geometrically Nonlinear Analysis

4.1 General information

A geometrically nonlinear analysis (GNA) can account for pre-buckling deformations. Unlike an eigenvalue buckling analysis, a geometrically nonlinear analysis does not directly provide buckling loads or buckling mode shapes. In a geometrically nonlinear analysis, the load at which a structure becomes unstable can be identified by examining force – displacement curves at selected locations. The nonlinear formulation options available in LUSAS include:

- Total Lagrangian

- Updated Lagrangian
- Eulerian
- Co-rotational
- P-Delta

If no perturbation is introduced and the applied load does not, by itself, initiate buckling because of its direction or distribution, a *negative pivot* is reported in the log output when the applied load exceeds the buckling load. Negative pivots reported during the iterative solution before convergence can be disregarded.

Note: If the buckling load associated with a higher-order mode is exceeded, additional negative pivots may be reported.

To determine whether a particular element type can be used in a geometrically nonlinear analysis, refer to the *Element Summary Tables* in the *LUSAS Element Reference Manual* and check whether the relevant geometric nonlinearity options are ticked for that element.

4.2 Example – Setting up a GNIA

A single LUSAS model can contain multiple analyses, including linear elastic, eigenvalue, and nonlinear analyses. Each analysis is independent of the others. While a geometric imperfection is not required to perform a geometrically nonlinear analysis, LUSAS allows the deformed shape from one analysis to be used as the initial configuration for another. This capability is useful when initial imperfections need to be considered.

In this example, Analysis 2 is set up as a geometrically nonlinear imperfection analysis. The first buckling mode shape shown in Figure 5 is adopted as an initial imperfection shape. The buckling mode shape has a maximum normalised displacement of 1; therefore, a scale factor of 0.01 is applied to introduce a maximum initial imperfection of 0.01 m in the geometrically nonlinear imperfection analysis. Figure 6 shows the *Initial State* tab, where the buckling mode shape and the scale factor to be used are specified.

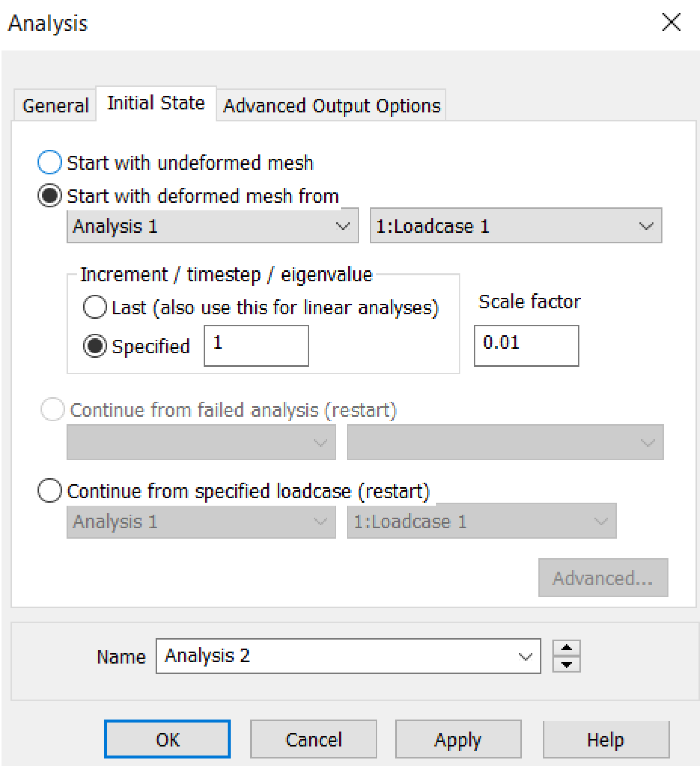

Figure 6 – Initial State tab.

To set up Analysis 2 as a nonlinear analysis:

- In the Analyses treeview, expand Analysis 2, right-click the loadcase listed under it, and select Controls > Nonlinear and Transient.
- Select the Nonlinear checkbox.

The *Nonlinear and Transient Controls* dialog is used to define the nonlinear analysis settings. In this example, the settings shown in Figure 7 are adopted. With the *Automatic Incrementation* option selected, the loads are applied incrementally.

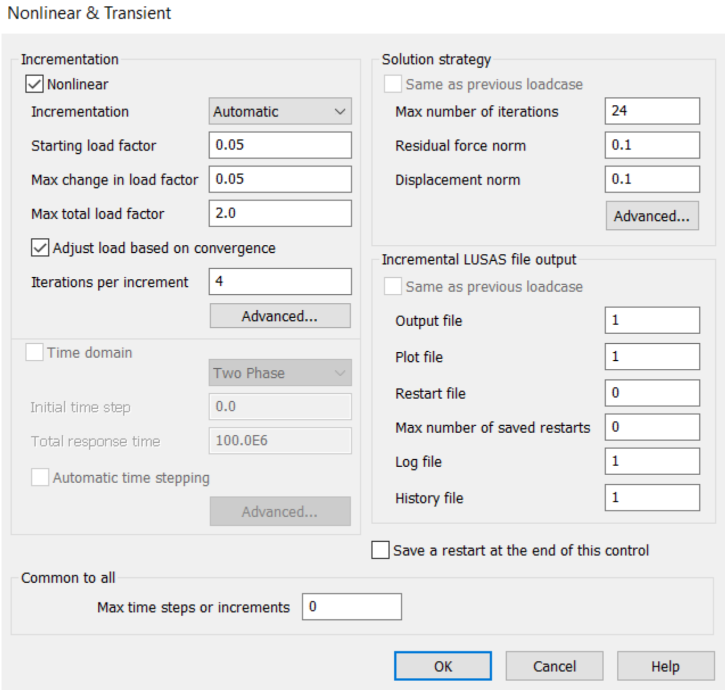

Figure 7 – Nonlinear & Transient Controls dialog.

To account for geometric nonlinearity, the *Total Lagrangian* and *Co-rotational formulation* options are selected in the *Nonlinear Analysis Options* dialog, as shown in Figure 8.

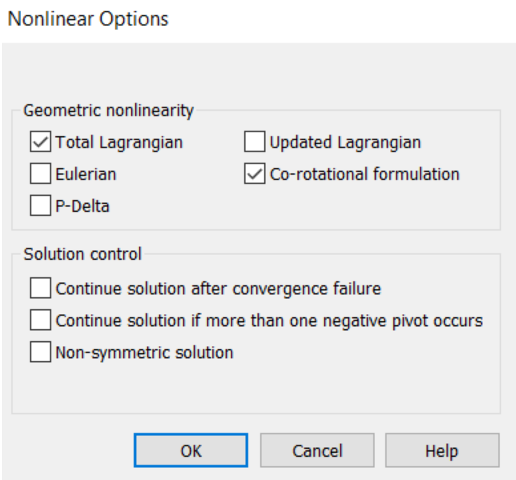

Figure 8 – Nonlinear Analysis Options dialog.

4.3 Example – Results

Figure 9 presents the relationship between the load factor and the lateral displacement at a node located at the midspan of one of the top flanges. The maximum load factor reached is 1.48, which is slightly lower than the eigenvalue buckling load factor of 1.60. However, a noticeable reduction in stiffness occurs at a load factor of approximately 1.40. Accordingly, a designer may consider adopting a design buckling load factor of 1.48 or lower. This highlights the importance of engineering judgement when interpreting the results of a geometrically nonlinear analysis.

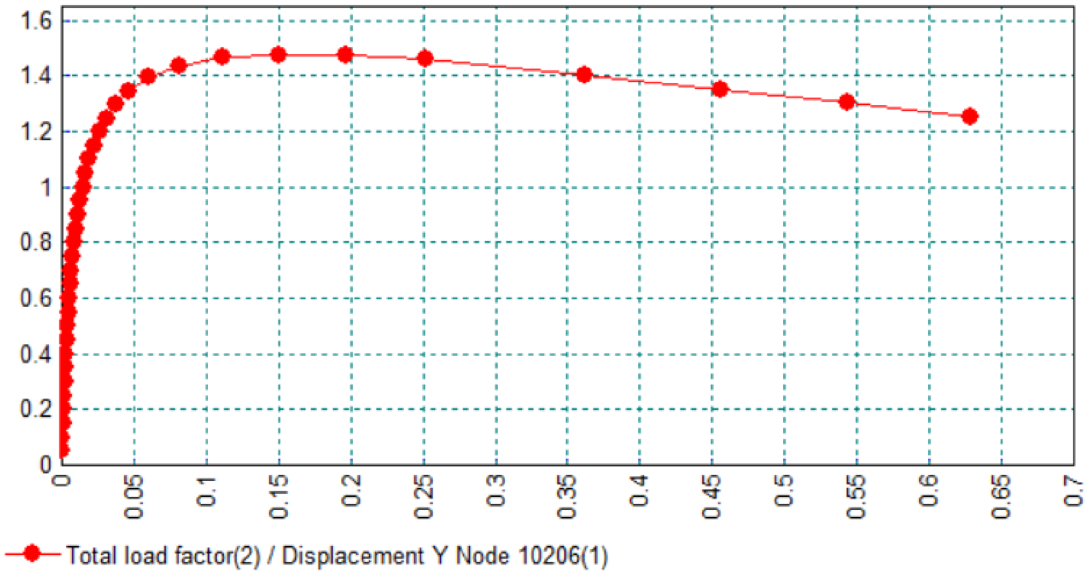

Figure 9 – GNIA: Total load factor vs lateral displacement graph.

5. Geometrically and Materially Nonlinear Analysis

5.1 General information

Material behaviour beyond the elastic range, such as yielding in steel and cracking in concrete, can be modelled using appropriate nonlinear stress–strain relationships. Material nonlinearity can be considered independently in a materially nonlinear analysis or combined with geometric nonlinearity in a geometrically and materially nonlinear analysis. Initial imperfections may also be incorporated, as discussed in the preceding section.

5.2 Example – Setting up a GMNIA

A copy of the geometrically nonlinear imperfection analysis (Analysis 2) described in the previous section can be used as the basis for the geometrically and materially nonlinear imperfection analysis (Analysis 3). Analysis 2 and Analysis 3 differ only in the material assigned to the shell elements representing the two plate girders.

In Analysis 2, an elastic material is assigned to the shell elements, whereas in Analysis 3, a nonlinear material model is used, with the plastic properties shown in Figure 10. For ductile materials such as steel, the von Mises yield criterion is generally appropriate. In addition to the initial uniaxial yield stress, a small strain-hardening gradient is introduced to improve numerical stability and aid convergence.

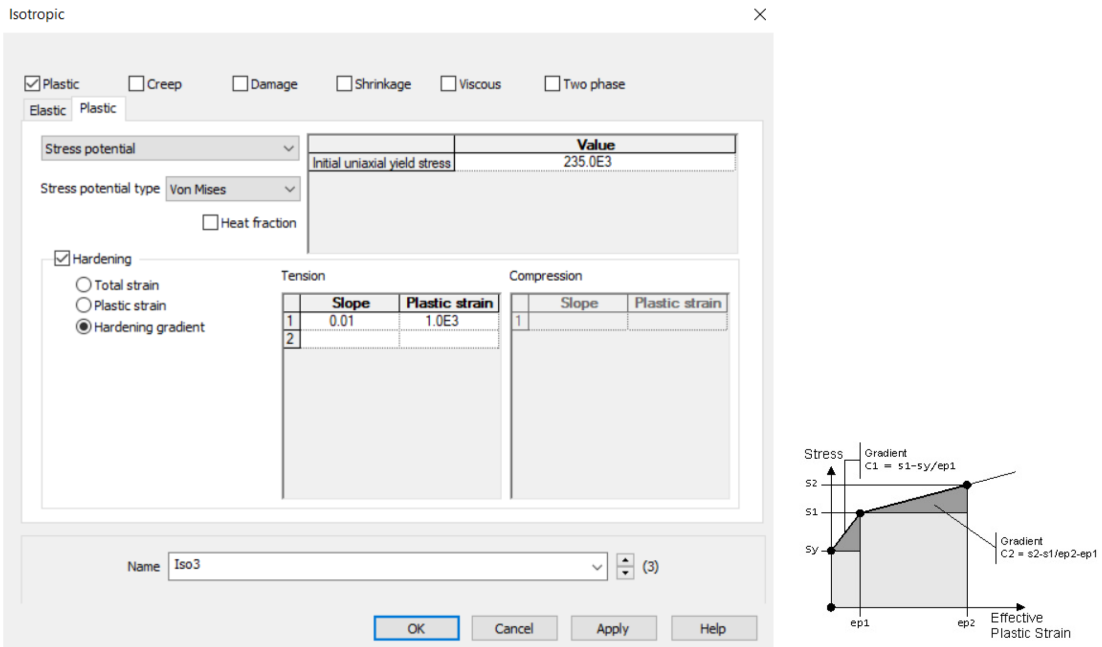

Figure 10 – Plastic properties of steel.

5.3 Example – Results

Figure 11 compares the results of the geometrically nonlinear imperfection analysis (GNIA) and the geometrically and materially nonlinear imperfection analysis (GMNIA). The blue curve represents the GNIA results, while the red curve represents the GMNIA results. Presenting both curves on the same graph clearly shows the effect of material nonlinearity on the structural response.

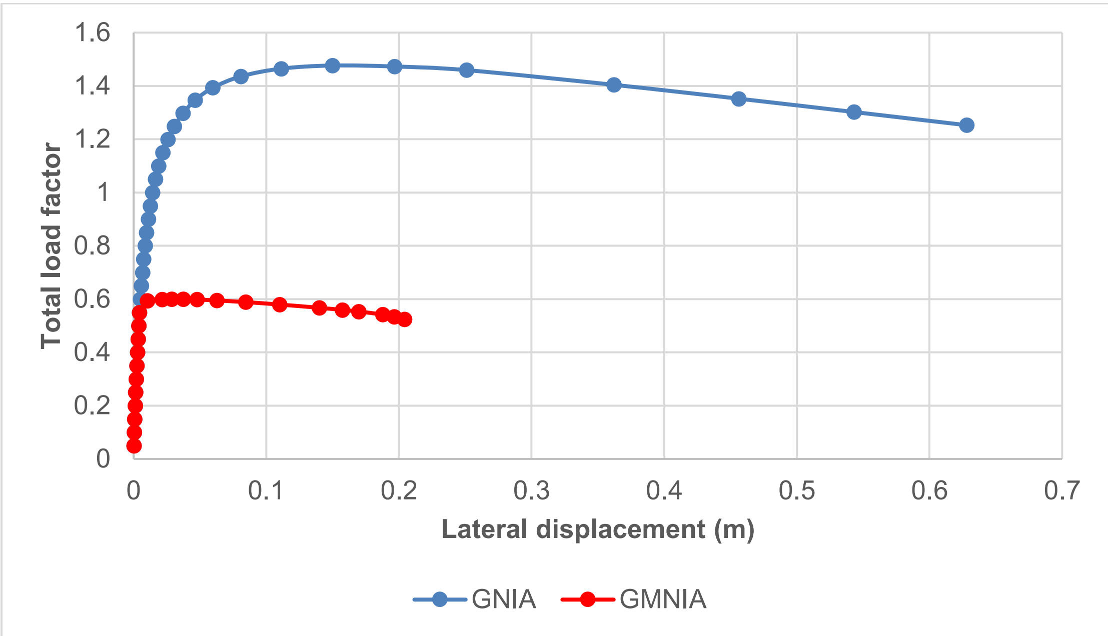

Figure 11 – Comparison between GNIA and GMNIA: Total load factor vs lateral displacement graph.

The Gauss points at which the von Mises stress reaches the material yield stress are indicated by red asterisks in Figure 12. As shown, significant yielding occurs in both the top and bottom flanges of the two plate girders around midspan.

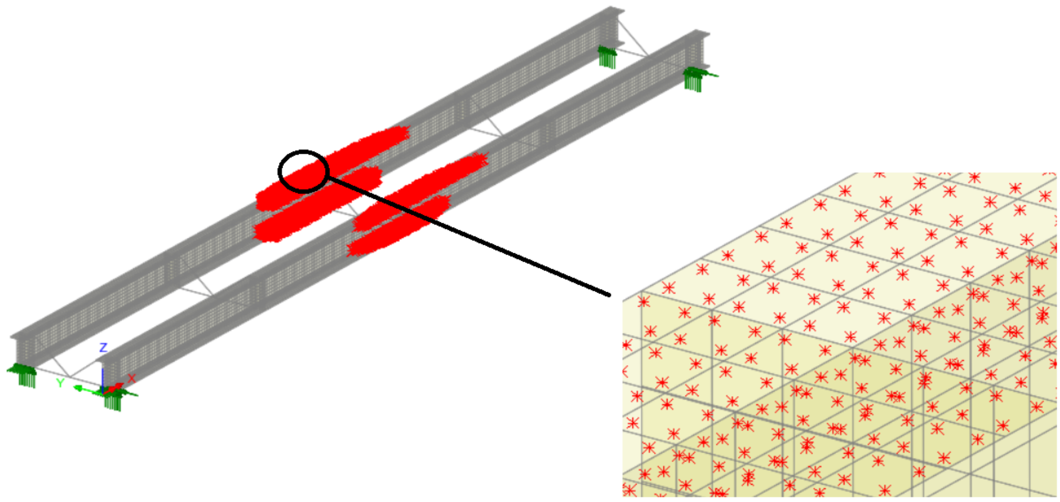

Figure 12 – Areas of yielding.

6. Discussion

This technical note provides a brief introduction to eigenvalue buckling analysis, geometrically nonlinear (imperfection) analysis, and geometrically and materially nonlinear (imperfection) analysis. A simple example is used to demonstrate how each of these three types of analysis can be carried out in LUSAS.

A geometrically and materially nonlinear imperfection analysis is generally expected to provide results that more closely reflect the actual behaviour of a structure. However, in addition to performing this type of analysis, conducting separate analyses in which each type of nonlinearity is considered independently can be valuable for identifying which type of nonlinearity has the greatest influence on the structural response.

A summary of the results is provided in Table 1. The influence of material nonlinearity is significant, as the failure load factor predicted by the geometrically nonlinear imperfection analysis (approximately 1.48) is substantially higher than that predicted by the geometrically and materially nonlinear imperfection analysis (approximately 0.60).

Method of analysis	Buckling/Failure Load Factor
Linear (Eigenvalue) Buckling Analysis	1.60
Geometrically Nonlinear Imperfection Analysis	~ 1.48
Geometrically and Materially Nonlinear Imperfection Analysis	~ 0.60

Table 1 Summary of results.

If you have any doubts or require specific advice for your type of analysis, please contact the LUSAS Technical Support team at support@lusas.com.