Unconfined Flow through Embankment

Keywords

2D, Plane Strain Two Phase, Pore Pressure, Water Pressure Distribution.

Problem Description

This example examines the problem of unconfined flow through a layer and the position of the resulting phreatic surface. Consider a vertical embankment that is 10 m high, as seen in Figure 1. Initially, the water table is at three metres, the level of the toe of the embankment. The water table is raised to the model's ground surface on the left, and the evolution of the new phreatic surface is calculated.

Discretisation

The model uses plane strain two phase elements with quadrilateral shape and quadratic interpolation (QPN8P elements). It is assumed that the bottom boundary of the model is impermeable. Seepage is allowed through the right side of the model through appropriate hydraulic boundary conditions. Figures 1 and 2, illustrate the boundary conditions used in the model.

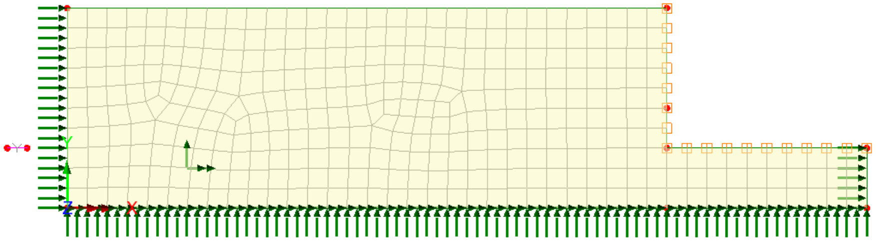


Figure 1: Mesh and boundary conditions

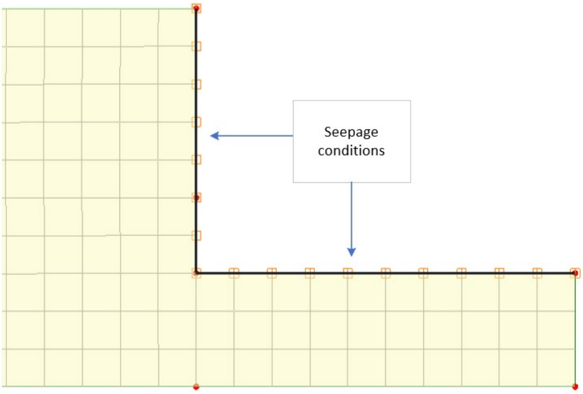


Figure 2: Hydraulic boundary conditions

Material Properties

Two-phase material properties are required when performing an analysis in which two phase elements are used to define a drained or undrained state for soil. Here in the following tables one and two, we can find the elastic and two-phase properties.

Table 1: Elastic material properties

Mass density	Young's modulus, E	Poisson's ratio, ν
1.5 t/m^3	750.0E3 kPa	0.25

Table 2: Two-phase material properties

Balk modulus of fluid phase	Porosity	Hydraulic conductivity (x/y/z direction)	Fluid Density	Van Ganuchten parameters	
				Rate of fluid extraction	Air entry
2.2E6 kN/m^2	0.5	0.1E-9 m/s	1 t/m^3	1.6	2.1 /m

Loading Conditions

In addition to gravity, a prescribed displacement is used to raise the water level from 3m to 10m (toe level to top of the embankment).

Theory

In unconfined flow the shape of the water table (phreatic surface) determines the flow distribution. If there is no recharge or evaporation, the quantity of water flowing in through the left side (upstream end) is equal to that flowing out through the right side (downstream end). For steady unconfined flow without recharge or evapotranspiration, Dupuit's equation gives the flow between two vertical cross sections

$$q = k\frac{h_1^2 - h_2^2}{2L} \qquad (1)$$

Where k is the hydraulic conductivity, L is the distance between sections and h_1 and h_2 the hydraulic heads at the first and second sections respectively.

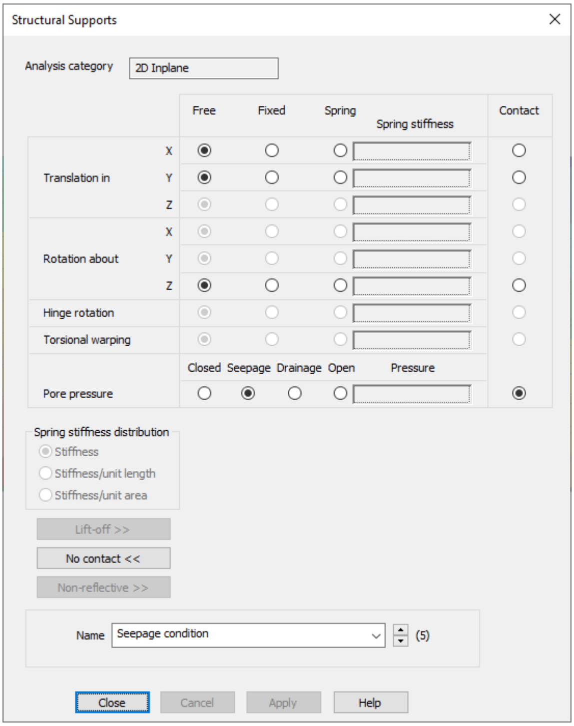
For steady one-dimensional unconfined flow in the absence of recharge or evapotranspiration, Dupuit parabola equation for the water table position is:

$$h(x) = [h_1^2 - \frac{(h_1^2 - h_2^2)\, x}{L}]^{0.5} \qquad (2)$$

Modelling Hints

A phreatic surface attribute is assigned to the line on the left of the model to define the position of the water table on the upstream side. It is marked with a Y. The phreatic surface is associated to the upstream face of the model using the water pressure distribution load type. Prescribed displacements applied to the line are used to raise the groundwater level. On the downstream side, seepage boundary conditions are applied to the embankment wall and floor.

Seepage boundary conditions are applied through the commands [Attributes > Support] (Figure 3)



Analysis category: 2D Inplane

		Free	Fixed	Spring	Spring stiffness	Contact
Translation in	X	●	○	○		○
	Y	●	○	○		○
	Z	●	○	○		○
Rotation about	X	●	○	○		○
	Y	●	○	○		○
	Z	●	○	○		○
Hinge rotation		●	○	○		○
Torsional warping		●	○	○		○

	Closed	Seepage	Drainage	Open	Pressure	Contact
Pore pressure	○	●	○	○		●

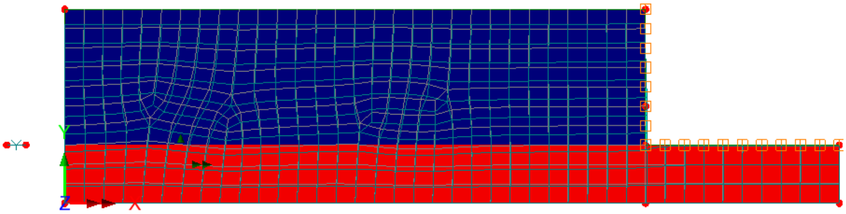
Spring stiffness distribution
● Stiffness
○ Stiffness/unit length
○ Stiffness/unit area

Name: Seepage condition (5)

Figure 3: Hydraulic boundary conditions

Comparison

The following figures 4 and 5, show the agreement between analytical solution using the equation 2 and LUSAS.



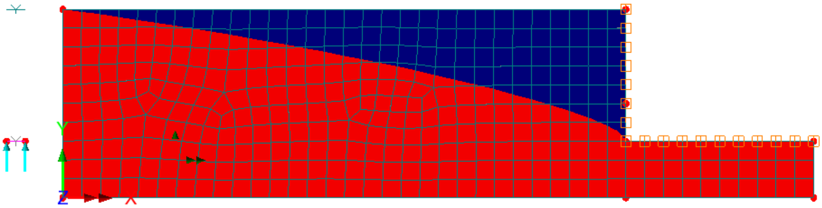


Figure 4: Water table at initial and final stage

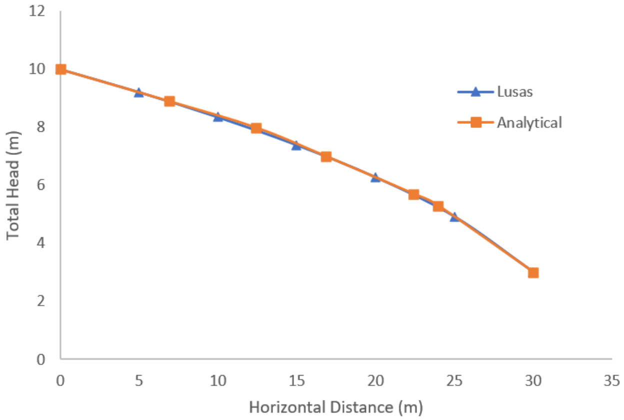
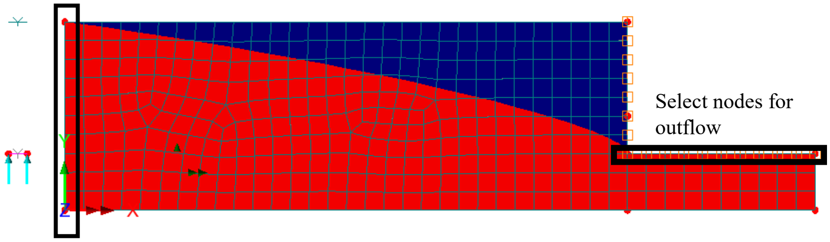


Figure 5: Comparison of total head (analytical, LUSAS)

The flow through the embankment can calculated by the employment of equation 1 between the vertical sections at 0 and 30m. By selecting all the nodes on the upstream boundary, the flow through the section can be plotted on a graph by selecting ‘reaction/VFlo’ and summing the nodal values. The outflow is similarly calculated by selecting all the nodes on the floor next to embankment wall.



Select nodes for outflow

Select nodes for inflow

Figure 6: Selection of nodes for calculation of inflow and outflow

Analytical	LUSAS Inflow	LUSAS Outflow
1.52×10^{-10} m^3/s	-1.50×10^{-10} m^3/s	1.50×10^{-10} m^3/s

References

[1] Fetter, C.W., Applied Hydrogeology. Third Edition, Prentice Hall, NJ, 1994.

Input Data

unconfined_flow.lvb